PHYSICAL REVIEW B **79**, 104434 (2009)

Theoretical study of the torque curves in some magnetic multilayer systems

A. Layadi
Département de Physique, Université Ferhat Abbas, Sétif 19000, Algeria
(Received 22 October 2008; published 30 March 2009)

Analytical expressions involving features from torque curves are derived for an exchange coupled [(AF)/(F)] bilayer, a coupled trilayer [(F)/(N-M)/(F)], and a magnetic tunnel junction (MTJ)-like [(AF)/(F)/(N-M)/(F)] system. Included in the model are the exchange anisotropy field H_E, the off-alignment angle β, the bilinear J_1, and the biquadratic J_2 coupling strengths and the in-plane magnetocrystalline anisotropy fields of the two layers whose easy axes make an angle δ. It will be shown how these parameters can be analytically derived from the torque curve. For the strong coupling case the MTJ-like system behaves as (AF)/(F) bilayer; for arbitrary δ value, the magnetic anisotropy field of the resulting system has a nonlinear relationship with the individual layer anisotropy fields.

DOI: 10.1103/PhysRevB.79.104434 PACS number(s): 75.30.Gw, 75.70.Cn

I. INTRODUCTION

A magnetic tunnel junction (MTJ)-like system (spin valve) has been lately the subject of a lot of experimental and theoretical studies because, among other reasons, of its use as giant magnetoresistive head in magnetic recording. Such a system may consist of a stacking of an antiferromagnetic (AF) thin film and two ferromagnetic (F) layer thin films (noted here A and B) separated by a nonmagnetic (N-M) interlayer[1–3] (see Fig. 1). The interaction at the interface of layer (A) with (AF) gives rise to a unidirectional anisotropy called exchange anisotropy.[4–16] This anisotropy can be modeled as a magnetic field $\mathbf{H}_E$, the exchange anisotropy field. It is also found that in some cases, the unidirectional anisotropy axis and the anisotropy axis of the ferromagnetic layer (A) are not parallel but make an angle β known as the off-alignment angle.[12–16] Layers (A) and (B) are magnetically coupled. The magnetic coupling can be described by the bilinear J_1 and the biquadratic J_2 coupling parameters.[17–26] The former may favor a parallel alignment (ferromagnetic coupling) or an antiparallel alignment (antiferromagnetic coupling) of the magnetizations $\mathbf{M}_A$ and $\mathbf{M}_B$ while the latter one (J_2) may lead to a perpendicular configuration of $\mathbf{M}_A$ and $\mathbf{M}_B$.

In previous work, ferromagnetic resonance (FMR) modes[27] and analytical expressions for the hysteresis curves[28] were investigated for such a system. In the present work some features of the torque curves are studied for this system; included in the model are the effects of the magnetic coupling strength, the exchange anisotropy field (H_E), the off-alignment angle β, and the uniaxial anisotropy fields (H_A and H_B) of the two ferromagnetic layers, with their easy axes making an angle δ between them.

One of the objectives of the present work is to derive analytical formulas giving the pertinent magnetic parameters of the system such as H_E, β, δ, and the interlayer magnetic coupling, as a function of some experimentally measurable quantities from torque curves. This may constitute an alternative method for a straightforward derivation of these parameters to the method based on the best fit as is usually done. Incidentally, some interesting behaviors of the system, at a relatively more fundamental level, are encountered and will be discussed. Some of these results confirm the ones inferred from FMR analysis[27] and from hysteresis curves.[28] The geometry of the system and the energy will be displayed in Sec. II. Some pertinent torque relations will be derived in Sec. III. The effect of different parameters will be discussed in Sec. IV for single layer, exchange [(F)/(AF)] bilayer, a trilayer [(F)A/N-M/(F)B] coupled system and a MTJ-like (or spin valve) system.

II. GENERAL CONSIDERATIONS

The magnetic systems under study in the present work are shown in Fig. 1. The more general one is the MTJ or spin valve. It consists of an antiferromagnetic layer (AF) on top of which is deposited a first ferromagnetic (F) layer (labeled A); layer A is separated from a second ferromagnetic layer B

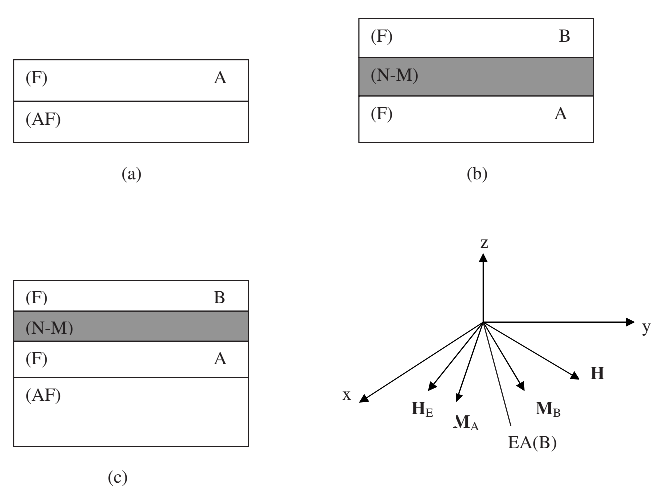

FIG. 1. The configuration of the three systems along with the coordinate system (a) exchange (F)/(AF) bilayer, (b) coupled trilayer, and (c) magnetic tunnel junction or spin valve structure. (F) ferromagnetic, (AF) antiferromagnetic and (N-M) nonmagnetic layers. $\mathbf{H}_E$ exchange anisotropy field, $\mathbf{H}$ applied magnetic field, $\mathbf{M}_A$ ($\mathbf{M}_B$): magnetization of layer A(B) and EA(B) layer B in-plane anisotropy easy-axis direction. All the vectors are in the film planes (xOy plane). The angles between the x axis and $\mathbf{H}$, $\mathbf{H}_E$, EA(B), $\mathbf{M}_A$ and $\mathbf{M}_B$ are denoted respectively α, β, δ, ϕ_A, and ϕ_B.

by a nonmagnetic (N-M) interlayer [see Fig. 1(c)]. All the thin film layers are assumed to lie in the x-y plane, with the z axis normal to the film planes (Fig. 1). The magnetization $\mathbf{M}_{\mathrm{A}}$ of layer A is defined, in spherical coordinates, by the angles θ_{A} and ϕ_{A}; and similarly $\mathbf{M}_{\mathrm{B}}$ (layer B) by the angles θ_{B} and ϕ_{B}. Two magnetic phenomena arise in this system:

(i) First, the exchange anisotropy at the interface antiferromagnetic-A layers, this is modeled as a magnetic field $\mathbf{H}_E$, the exchange anisotropy field. The field $\mathbf{H}_E$ is taken in the film plane making an angle β (the off-alignment angle) with the x axis; layer A is supposed to have an in-plane uniaxial magnetocrystalline anisotropy with constant K_{A} and the easy axis taken to be along the x axis (see Fig. 1).

(ii) Second, the magnetic coupling between two ferromagnetic layers separated by a nonmagnetic interlayer. This interaction will be described by the bilinear J_1 and biquadratic J_2 coupling parameters. For layer (B), the in-plane magnetocrystalline anisotropy easy axis (EA) is assumed to be in a direction making an angle δ with the x axis (Fig. 1).

The external applied magnetic field $\boldsymbol{H}$ is taken to be in the plane of the films, making an angle α with the x axis. At equilibrium, the magnetizations $\mathbf{M}_{\mathrm{A}}$ and $\mathbf{M}_{\mathrm{B}}$ are expected to lie in the film plane, i.e., $\theta_{\mathrm{A}}=\theta_{\mathrm{B}}=\pi/2$, due to the strong demagnetizing field of the thin films and to the fact that the applied magnetic field is in plane. Note that contrary to ultrathin films consisting of one or a few atomic planes where it was observed that the magnetization can be perpendicular to the film plane due to a strong positive magnetocrystalline[29] (or surface) anisotropy, may oscillate from in plane to out of plane[30,31] as the thickness varies or may be tilted,[32] here it is assumed that the films are in the thickness range where the shape anisotropy dominates and the magnetizations lie in the film planes.

With all these considerations, the total free energy of the system per unit area can be explicitly written as

$$\begin{aligned} E = {} & t_{\mathrm{A}}[-M_{\mathrm{A}}H\cos(\alpha-\phi_{\mathrm{A}})+K_{\mathrm{ueffA}} \\ & -K_{\mathrm{A}}\cos^2\phi_{\mathrm{A}}-M_{\mathrm{A}}H_E\cos(\beta-\phi_{\mathrm{A}})] \\ & +t_{\mathrm{B}}[-M_{\mathrm{B}}H\cos(\alpha-\phi_{\mathrm{B}})+K_{\mathrm{ueffB}}-K_{\mathrm{B}}\cos^2(\phi_{\mathrm{B}}-\delta)] \\ & -J_1\cos(\phi_{\mathrm{A}}-\phi_{\mathrm{B}})-J_2\cos^2(\phi_{\mathrm{A}}-\phi_{\mathrm{B}}). \end{aligned} \tag{1}$$

In the two first lines of Eq. (1), t_{A} and t_{B} are the thicknesses of layers A and B, respectively. The total energy E consists for layer A (the first line) of the Zeeman energy (interaction of the external magnetic field $\mathbf{H}$ with the magnetizations), the shape and any out-of-plane uniaxial anisotropy with effective constant K_{ueffA} ($K_{\mathrm{ueffA}}=K_{\mathrm{u}}-2\pi M_{\mathrm{A}}^2$, K_{u} being the uniaxial magnetocrystalline anisotropy constant), the in-plane magnetocrystalline anisotropy with constant K_{A} and the exchange anisotropy with exchange anisotropy field H_E. For layer B [the second line in Eq. (1)], the Zeeman energy, the effective uniaxial, and in-plane anisotropy terms are displayed. The interlayer coupling energy is given by the two last terms [third line in Eq. (1)]. The angles $\phi_{\mathrm{A,B}}$ are given by the following two coupled equations (the equilibrium conditions):

$$\begin{aligned} H\sin(\alpha-\phi_{\mathrm{A}}) = {} & \frac{H_{\mathrm{A}}}{2}\sin 2\phi_{\mathrm{A}}-H_E\sin(\beta-\phi_{\mathrm{A}}) \\ & +\frac{J_1}{a}\sin(\phi_{\mathrm{A}}-\phi_{\mathrm{B}})+\frac{J_2}{a}\sin 2(\phi_{\mathrm{A}}-\phi_{\mathrm{B}}) \end{aligned} \tag{2a}$$

and

$$\begin{aligned} H\sin(\alpha-\phi_{\mathrm{B}}) = {} & \frac{H_{\mathrm{B}}}{2}\sin 2(\phi_{\mathrm{B}}-\delta)-\frac{J_1}{b}\sin(\phi_{\mathrm{A}}-\phi_{\mathrm{B}}) \\ & -\frac{J_2}{b}\sin 2(\phi_{\mathrm{A}}-\phi_{\mathrm{B}}), \end{aligned} \tag{2b}$$

where $H_{\mathrm{A}}=2K_{\mathrm{A}}/M_{\mathrm{A}}$ and $H_{\mathrm{B}}=2K_{\mathrm{B}}/M_{\mathrm{B}}$ are the planar anisotropy fields for layers A and B, respectively, and $a=t_{\mathrm{A}}M_{\mathrm{A}}$ and $b=t_{\mathrm{B}}M_{\mathrm{B}}$.

III. GENERAL TORQUE RELATIONS

In the following, expressions for the torque curves and the slope of the curve at any point will be derived and discussed as a function of the different parameters: the exchange and anisotropies fields, the β and δ angles and the magnetic coupling strengths.

The torque relation can be obtained from the total energy in Eq. (1). The torque, per unit area, is found to be

$$T=\frac{aH_{\mathrm{A}}}{2}\sin 2\phi_{\mathrm{A}}+aH_E\sin(\phi_{\mathrm{A}}-\beta)+\frac{bH_{\mathrm{B}}}{2}\sin 2(\phi_{\mathrm{B}}-\delta). \tag{3}$$

The angles ϕ_{A} and ϕ_{B} are given by Eqs. (2a) and (2b).

The torque curve slope, s, is defined as the derivative of the torque with the respect to the applied field angle α, i.e., $s=\frac{dT}{d\alpha}$. Torque curve slopes have been used to study the magnetic anisotropy in single thin films,[33] the magnetic coupling in a coupled layer system (ferromagnetic/nonmagnetic/ferromagnetic)[34] and the exchange anisotropy field in bilayer exchanged thin films (ferromagnetic/antiferromagnetic).[35]

The torque T in Eq. (3) depends explicitly on the angles ϕ_{A} and ϕ_{B} and implicitly on the applied field angle α. The derivative of T with respect to α will then be found by computing the derivatives of T with respect to ϕ_{A} and ϕ_{B} from Eq. (3) and the derivatives of ϕ_{A} and ϕ_{B} with respect to α which can be found from Eqs. (2a) and (2b). This can be done by taking the total variation of both sides of Eqs. (2a) and (2b), dividing by $d\alpha$, and solving the two resulting equations for the derivatives of ϕ_{A} and ϕ_{B} with respect to α. Following this method, one can show that the slope is given by the expression

$$s = \frac{H[aX_B Y_A \cos(\alpha-\phi_A) + bX_A Y_B \cos(\alpha-\phi_B) + J_{\text{eff}} e(Y_A+Y_B)]}{X_A X_B + J_{\text{eff}}(X_A+X_B)}, \tag{4}$$

where

$$X_A = a[H\cos(\alpha-\phi_A) + H_A\cos 2\phi_A + H_E\cos(\beta-\phi_A)], \tag{5a}$$

$$X_B = b[H\cos(\alpha-\phi_B) + H_B\cos 2(\phi_B-\delta)], \tag{5b}$$

$$Y_A = a[H_A\cos 2\phi_A + H_E\cos(\beta-\phi_A)], \tag{5c}$$

$$Y_B = bH_B\cos 2(\phi_B-\delta), \tag{5d}$$

$$e = a\cos(\alpha-\phi_A) + b\cos(\alpha-\phi_B), \tag{5e}$$

and the effective magnetic coupling parameter

$$J_{\text{eff}} = J_1\cos(\phi_A-\phi_B) + 2J_2\cos 2(\phi_A-\phi_B). \tag{5f}$$

IV. DISCUSSIONS

A. Single and exchange-coupled layers

When the layers (A) and (B) are uncoupled, $J_1=J_2=0$. If the two layers were separate, then experimentally, one obtains either the curves corresponding to (AF)/(A) [Fig. 2(a)] or those of the single (B) layer (Fig. 3). When the two ferromagnetic layers are uncoupled in a spin valve system, then one gets the sum of the two corresponding curves. For the latter case, the slope of the torque curve at any point is given by substituting $J_{\text{eff}}=0$ in Eq. (4). Replacing the different quantities by their values [Eqs. (5a)–(5e)], one finds

$$s(J_{\text{eff}}=0) = \frac{aH[H_A\cos 2\phi_A + H_E\cos(\beta-\phi_A)]\cos(\alpha-\phi_A)}{H\cos(\alpha-\phi_A) + H_A\cos 2\phi_A + H_E\cos(\beta-\phi_A)} + \frac{bHH_B\cos 2(\phi_B-\delta)\cos(\alpha-\phi_B)}{H\cos(\alpha-\phi_B) + H_B\cos 2(\phi_B-\delta)}. \tag{6}$$

The slope given by Eq. (6) is just the sum of the slopes of the torque curves corresponding to the (A) layer with off-aligned exchange anisotropy (the first term) and to layer (B) with tilted in-plane uniaxial anisotropy axis.

The (AF)/(A) curves shown in Fig. 2(a) correspond to the aligned case, $\beta=0$ (solid line) and the off-aligned one, $\beta \neq 0$ (dashed line). These are typical curves for the system with exchange anisotropy, i.e., (i) they have only one stable position, (ii) the shift of the curve (dashed one) is due to the off-alignment. It is easy to relate the shift α_0 in the torque curve (the point where the curve crosses the α axis) to the off-alignment β angle. Figure 2(b) presents an example of the variation in the slope at $\alpha_0=0$ (for $\beta=0$, solid line) and at α_0 (for $\beta=20°$, dashed line) with the exchange anisotropy field H_E; as one can see the variation in the slope is quite clear and easily detectable. Also the variation in the shift α_0 with H_E is shown in this figure as a dotted line.

From an experimental torque curve, on can measure the shift α_0, the slope s_0 of the curve at α_0, the maximum T_m of the curve and the angle α_m where the maximum occurs. Recall that the shift or crossing point α_0 is given by $T(\alpha_0)=0$ which is achieved by the condition $\alpha_0=\phi_A$ and leads [from the A part of the torque, i.e., the two first terms of Eq. (3)] to the following equation:

$$\frac{H_A}{2}\sin 2\alpha_0 = H_E\sin(\beta-\alpha_0). \tag{7a}$$

From the first part [corresponding to the (AF)/(A) system] of Eq. (6), the slope s_0 at α_0 will then be given by

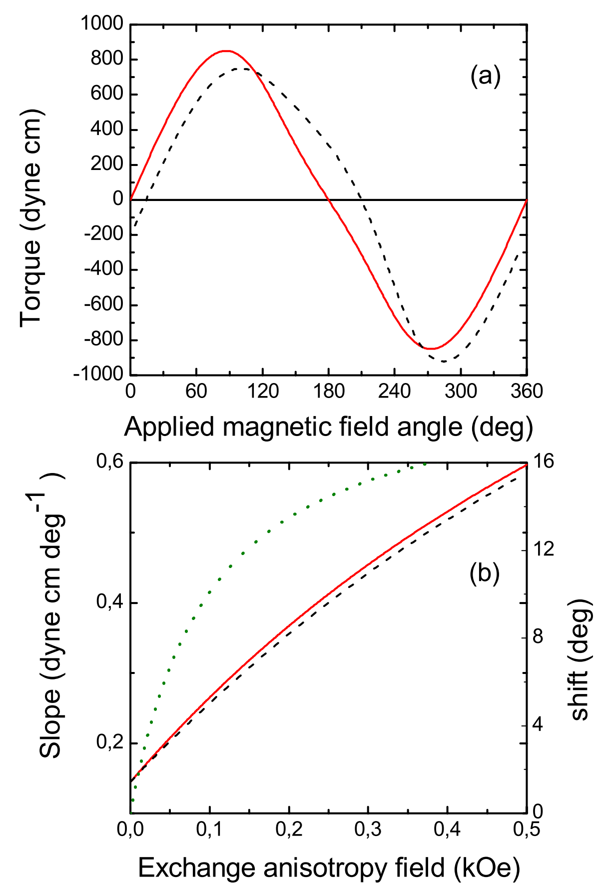

FIG. 2. (Color online) (a) Torque vs applied field angle for an (AF)/(F) exchanged coupled layers [layer (A) here], with $H_E=0.25$ kOe and the aligned case $\beta=0$ (solid line) and the off-aligned case $\beta=20°$ (dashed line) (b) Variation in the slope (solid line for $\beta=0$ and dashed line for $\beta=20°$) and the shift α_0 (dotted line) with the exchange anisotropy field H_E. Other parameters used in the computation: $4\pi M_A=10$ kG, $H_A=0.1$ kOe, $t_A=20$ nm. Applied magnetic field $H=1$ kOe.

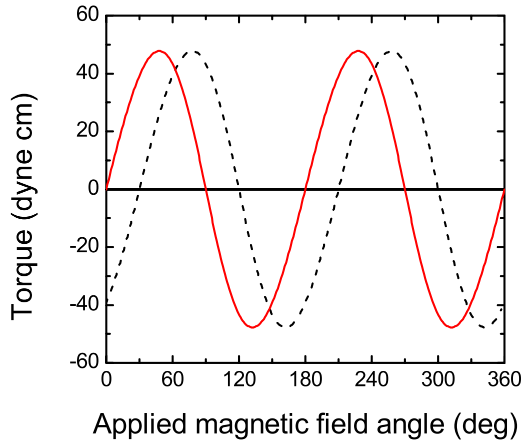

FIG. 3. (Color online) Torque vs applied field angle for a single ferromagnetic layer with in-plane magnetocrystalline anisotropy [layer (B)] with $H_B=0.1$ kOe and tilt angle $\delta=0$ (solid line), $\delta=30°$ (dashed line). Other parameters used in the computation: $4\pi M_B=6$ kG, $t_B=10$ nm. Applied magnetic field $H=1$ kOe.

$$s_0=\frac{aH[H_A\cos 2\alpha_0+H_E\cos(\beta-\alpha_0)]}{H+H_A\cos 2\alpha_0+H_E\cos(\beta-\alpha_0)}. \tag{7b}$$

The maximum T_m of the curve occurs at $\alpha=\alpha_m$ such that $s(\alpha_m)=0$ whose solution can be $\phi_A=\alpha_m-\pi/2$ (considering the first branch without loss of generality). Inserting this solution in the A part of the torque [the two first terms in Eq. (3)] will lead to the following expression for T_m:

$$T_m=-\frac{aH_A}{2}\sin 2\alpha_m-aH_E\cos(\alpha_m-\beta). \tag{7c}$$

After some algebraic manipulations, the following analytical relations are found for the in-plane anisotropy field H_A, the exchange anisotropy field H_E, and the off-alignment angle β:

$$H_A=\frac{1}{F}\left[\frac{Hs_0}{s_0-aH}\cos(\alpha_m-\alpha_0)-\frac{T_m}{a}\right], \tag{8}$$

where F is a function of the angles $(\alpha_0,\ \alpha_m)$ and is given by $[\sin\alpha_m\cos\alpha_m+\sin^3\alpha_0\sin\alpha_m-\cos^3\alpha_0\cos\alpha_m]$. In order to derive Eq. (8), one has to express $H_E\cos(\alpha_m-\beta)$ in Eq. (7c) in terms of H_A and measurable quantities from torque curve (s_0, α_0, α_m, and T_m). For that, one has first to make use of a trigonometric relation which gives $H_E\cos(\alpha_m-\beta)=H_E\sin(\beta-\alpha_0)\sin(\alpha_m-\alpha_0)+H_E\cos(\beta-\alpha_0)\cos(\alpha_m-\alpha_0)$. Then one can find $H_E\sin(\beta-\alpha_0)$ and $H_E\cos(\beta-\alpha_0)$ from Eqs. (7a) and (7b), respectively. Substituting these expressions into Eq. (7c) and solving for H_A, one will get Eq. (8).

Having found H_A from Eq. (8), H_E will then be given, in terms of T_m, H_A, α_0, and α_m by

$$H_E=\frac{1}{\cos(\alpha_m-\alpha_0)}\left[\frac{T_m^2}{a^2}+\frac{H_A^2}{4}(G^2+I^2)+\frac{T_mH_AG}{a}\right]^{1/2}, \tag{9a}$$

where G and I are trigonometrical functions of α_0 and α_m and are given, respectively, by $[\sin 2\alpha_m+\sin 2\alpha_0\sin(\alpha_m-\alpha_0)]$ and $[\sin 2\alpha_0\cos(\alpha_m-\alpha_0)]$. Alternatively H_E can be obtained using α_0 and s_0 only by:[15]

$$H_E=\frac{1}{2}\left[H_A^2\sin^2 2\alpha_0+4\left(\frac{s_0H}{aH-s_0}-H_A\cos 2\alpha_0\right)^2\right]^{1/2}. \tag{9b}$$

With the knowledge of H_E and H_A, the off-alignment angle β will then be given by

$$\beta=\alpha_0+\sin^{-1}\left[\frac{H_A\sin 2\alpha_0}{2H_E}\right]. \tag{10}$$

Note that in the aligned case, where $\beta=0$, the shift is equal to zero [see Eq. (10)] and Eqs. (9a) and (9b) reduce simply to the following forms:

$$H_E=\frac{1}{\cos\alpha_m}\left[\frac{T_m}{a}+\frac{H_A\sin 2\alpha_m}{2}\right] \tag{11}$$

and

$$H_E=\frac{s_0H}{aH-s_0}-H_A. \tag{12}$$

Thus with the measure of the shift α_0, the slope s_0 of the curve at α_0, the maximum T_m of the curve and the angle α_m where the maximum occurs, one can obtain in a straightforward and unique manner the in-plane anisotropy field H_A, the exchange anisotropy field H_E, and the off-alignment angle β. This analysis could be useful for investigating (AF)/(F) exchange bilayer thin film.

The curve corresponding to layer (B) (Fig. 3) is that of a single ferromagnetic layer with in-plane magnetic anisotropy with (i) the easy direction along the x axis (the easy directions of layers A and B are parallel) (solid line), (ii) an arbitrary easy-axis direction given by the tilt angle δ (dashed line). The latter curve is shifted due to the tilt angle δ; if $\delta=0$, then the curve will cross the α axis at $\alpha_B=0$ (solid line). Here, it is easy to show that the shift angle α_B is simply equal to the tilt angle δ. Moreover the measure of the torque curve slope, at $\alpha=\alpha_B$, $s_B(\alpha_B)$ will lead to the value of the in-plane anisotropy field intensity H_B:

$$H_B=\frac{s_B(\alpha_B)H}{t_BM_BH-s_B(\alpha_B)}. \tag{13}$$

B. Strongly coupled MTJ system

The other extreme case, i.e., very strongly coupled layers will now be considered. When the coupling is ferromagnetic, then the two magnetizations $\mathbf{M}_A$ and $\mathbf{M}_B$ are always parallel under the strong magnetic coupling, i.e., $\phi_A=\phi_B=\phi_F$ for all α angles. The torque will then be given by Eq. (3) substituting the magnetizations angles by ϕ_F. For the torque curve slope, one may keep only the J_{eff} term in Eq. (4). After some algebraic manipulations, Eq. (4) will then reduce to the following form for the slope s_∞ (F) (for infinite ferromagnetic coupling):

$$s_\infty(\mathrm{F}) = \frac{(a+b)H[H_p^{\mathrm{F}} \cos 2(\phi_{\mathrm{F}} - \delta^{\mathrm{eq}}) + H_E^{\mathrm{F}} \cos(\beta^{\mathrm{eq}} - \phi_{\mathrm{F}})]\cos(\alpha - \phi_{\mathrm{F}})}{H \cos(\alpha - \phi_{\mathrm{F}}) + H_p^{\mathrm{F}} \cos 2(\phi_{\mathrm{F}} - \delta^{\mathrm{eq}}) + H_E^{\mathrm{F}} \cos(\beta^{\mathrm{eq}} - \phi_{\mathrm{F}})} \tag{14a}$$

and the angle ϕ_{F} will be given by

$$H \sin(\alpha - \phi_{\mathrm{F}}) = \frac{H_p^{\mathrm{F}}}{2} \sin 2(\phi_{\mathrm{F}} - \delta^{\mathrm{eq}}) - H_E^{\mathrm{F}} \sin(\beta^{\mathrm{eq}} - \phi_{\mathrm{F}}). \tag{14b}$$

The torque curve corresponding to this strong ferromagnetic coupling is shown in Fig. 4 (solid line). Equations (14a) and (14b) [compare these equations to Eqs. (6) and (2)] as well as the torque curve for this strongly coupling case [compare the curves in Figs. 2(a) and 4] show that this strongly coupled MTJ system is equivalent to an (F)/(AF) exchanged bilayer thin film with effective in-plane and exchange anisotropy fields H_p^{F} and H_E^{F} and effective tilt δ^{eq} and off-aligned β^{eq} angles. These quantities are given respectively by

$$H_p^{\mathrm{F}} = \frac{\sqrt{(aH_{\mathrm{A}} + bH_{\mathrm{B}} \cos 2\delta)^2 + b^2 H_{\mathrm{B}}^2 \sin^2 2\delta}}{a+b}, \tag{15}$$

$$H_E^{\mathrm{F}} = \frac{aH_E}{a+b}, \tag{16}$$

$$\delta^{\mathrm{eq}} = \frac{1}{2}\tan^{-1}\left[\frac{bH_{\mathrm{B}} \sin 2\delta}{aH_{\mathrm{A}} + bH_{\mathrm{B}} \cos 2\delta}\right], \tag{17}$$

while $\beta^{\mathrm{eq}} = \beta$.

For the strong antiferromagnetic coupling, assuming $a > b$, the (A) magnetization will tend to align along the $\mathbf{H}$ direction while the (B) magnetization will be antiparallel to $\mathbf{M}_{\mathrm{A}}$; one has, for all α angles, $\phi_{\mathrm{A}} = \phi_{\mathrm{AF}}$ and $\phi_{\mathrm{B}} = \pi + \phi_{\mathrm{AF}}$. Following the same procedure as for the previous case, one finds that the slope s_∞ (AF) (for infinite antiferromagnetic coupling) can be written [in the same form as Eq. (14a)]

$$s_\infty(\mathrm{AF}) = \frac{(a-b)H[H_p^{\mathrm{AF}} \cos 2(\phi_{\mathrm{AF}} - \delta^{\mathrm{eq}}) + H_E^{\mathrm{AF}} \cos(\beta^{\mathrm{eq}} - \phi_{\mathrm{AF}})]\cos(\alpha - \phi_{\mathrm{AF}})}{H \cos(\alpha - \phi_{\mathrm{AF}}) + H_p^{\mathrm{AF}} \cos 2(\phi_{\mathrm{AF}} - \delta^{\mathrm{eq}}) + H_E^{\mathrm{AF}} \cos(\beta^{\mathrm{eq}} - \phi_{\mathrm{AF}})} \tag{18}$$

and the angle ϕ_{AF} is given by an equation similar to Eq. (14b) by substituting H_p^{F} and H_E^{F} by H_p^{AF} and H_E^{AF}, respectively. Here the effective in-plane anisotropy field H_p^{AF} is given by

$$H_p^{\mathrm{AF}} = \frac{\sqrt{(aH_{\mathrm{A}} + bH_{\mathrm{B}} \cos 2\delta)^2 + b^2 H_{\mathrm{B}}^2 \sin^2 2\delta}}{a-b} \tag{19}$$

and the effective exchange anisotropy field

$$H_E^{\mathrm{AF}} = \frac{aH_E}{a-b}. \tag{20}$$

The torque curve for this strong antiferromagnetic coupling is shown in Fig. 4 (dashed line). Once again this strongly (antiferromagnetic) coupled MTJ system is equivalent to an (F)/(AF) exchanged bilayer thin film with effective in-plane and exchange anisotropy fields H_p^{AF} and H_E^{AF} (different for those found in the ferromagnetic coupling case). The effective tilt angle is the same as in the ferromagnetic coupling case, i.e., $\delta^{\mathrm{AF}} = \delta^{\mathrm{F}} = \delta^{\mathrm{eq}}$ [given by Eq. (17)] and the effective off-aligned angles ($\beta^{\mathrm{AF}} = \beta^{\mathrm{F}}$) are equal to original off-aligned angle (β).

Note that there is a nonlinear relationship between the effective in-plane field for the strongly coupled MTJ system and the in-plane anisotropy fields for the individual layers [see Eqs. (15) and (19)]. Some particular cases are worth noting where the relations become linear. In the situation where the easy axes are parallel ($\delta = 0$), the effective in-plane anisotropy field reduces to the following:

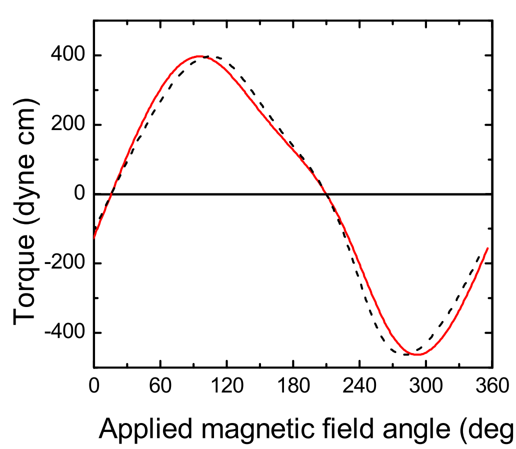

FIG. 4. (Color online) Torque curves for strongly coupled MTJ system with $\beta = 20°$ and $\delta = 30°$, for ferromagnetic (solid line) and antiferromagnetic (dashed line) coupling cases. Other parameters as for Figs. 2 and 3.

$$H_p^{\mathrm{eq}}(\delta=0)=\frac{aH_{\mathrm{A}}+bH_{\mathrm{B}}}{a\pm b} \tag{21}$$

and the tilt angle $\delta^{\mathrm{eq}}=0$, where + and − signs in Eq. (21) correspond to ferromagnetic and antiferromagnetic couplings, respectively. For orthogonal easy axes ($\delta=90°$), one obtains

$$H_p^{\mathrm{eq}}(\delta=90)=\frac{aH_{\mathrm{A}}-bH_{\mathrm{B}}}{a\pm b} \tag{22}$$

and $\delta^{\mathrm{eq}}=0$. In the case of the rigid layer model[36,37] where the in-plane anisotropy of one layer [layer (A)] is very strong compared to that of the other layer [layer (B)], one obtains

$$H_p^{\mathrm{eq}}=\frac{aH_{\mathrm{A}}+bH_{\mathrm{B}}\cos 2\delta}{a\pm b} \tag{23}$$

and

$$\delta^{\mathrm{eq}}=\frac{1}{2}\tan^{-1}\left[\frac{bH_{\mathrm{B}}\sin 2\delta}{aH_{\mathrm{A}}}\right]. \tag{24}$$

δ^{eq} in this case is close to zero, i.e., the easy axis of the whole system is along the layer (A) easy axis. Finally for the particular case where $aH_{\mathrm{A}}=bH_{\mathrm{B}}=Q$, then H_p^{eq} and δ^{eq} reduce to

$$H_p^{\mathrm{eq}}=\frac{2Q\cos\delta}{a\pm b} \tag{25}$$

and $\delta^{\mathrm{eq}}=\frac{\delta}{2}+\frac{n\pi}{2}$, where n is an integer.

Thus, for a strong interlayer magnetic coupling, and for both ferromagnetic (labeled FC) and antiferromagnetic (AFC) coupling, the whole [(AF)/(F)/N-M/(F)] magnetic tunnel junction system behaves as a bilayer [(AF)/(F)] thin film, i.e., a single ferromagnetic layer with an effective in-plane anisotropy field whose axis is tilted from the reference axis (the x axis) by an angle δ^{eq} and an effective exchange anisotropy field with an off-aligned angle β^{eq}. These quantities are expressed in terms of the individual layers parameters. Some remarks can be made about these quantities: (i) the effective off-aligned angle β^{eq} is the same for (FC) and (AFC) and is equal to the original β angle of layer (A), (ii) the effective tilt angle δ^{eq} is equal for (FC) and (AFC) and is different from δ, (iii) for arbitrary tilt angle δ the effective in-plane anisotropy field is given by a nonlinear relationship as a function of the individual anisotropy fields (H_{A} and H_{B}), one obtains a linear relationship only in some particular situations, (iv) the effective exchange anisotropy field is less than the original H_E field of layer (A) for (FC) while it is greater than H_E for (AFC), this enables one to increase H_E^{eq} to any value by the right choice of a and b (recall that $a=t_{\mathrm{A}}M_{\mathrm{A}}$ and $b=t_{\mathrm{B}}M_{\mathrm{B}}$). Some of these results confirm those found in these systems by ferromagnetic resonance[27] and magnetization curves.[28]

Also for this strong coupling case an interesting relation is found between the slopes for the FC and the AFC cases at the crossing point. Note that the crossing point (the intersection of the torque curve with the α axis) is the same for both cases (see Fig. 4); this is because the effective tilt and off-alignment angles are the same. In fact, the crossing point α_0^*

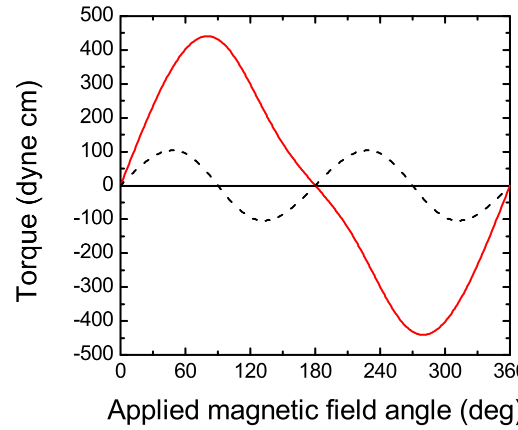

FIG. 5. (Color online) Torque curves for coupled MTJ system (solid line) and coupled trilayer system [(F)A/nonmagnetic/(F)B] (dashed line). $H_E=0.25$ kOe, $\beta=0°$, and $\delta=0°$. Other parameters as for Figs. 2 and 3.

for the strong FC and AFC is found to satisfy the following relation:

$$\frac{\sin 2(\alpha_0^*-\delta^{\mathrm{eq}})}{\sin(\beta-\alpha_0^*)}=\frac{2aH_E}{\sqrt{(aH_{\mathrm{A}}+bH_{\mathrm{B}}\cos\ 2\delta)^2+b^2H_{\mathrm{B}}^2\sin^2 2\delta}}. \tag{26}$$

It is easy to see that this general relation [Eq. (26)] reduces correctly to Eq. (10) for an exchange coupled bilayer thin film [layer (A) only] and to the fact that the shift is equal to the tilt angle for layer (B) only. However the slopes s_∞ (F) and s_∞ (AF) are different at the crossing point. The following relation is found:

$$\frac{1}{s_\infty(\mathrm{AF})}-\frac{1}{s_\infty(\mathrm{F})}=\frac{2b}{H(a^2-b^2)}. \tag{27}$$

It is interesting to note that the difference in the inverse slopes does not depend on the anisotropies (exchange and in-plane) and on the crossing point. It depends only on the applied field H, the thickness and magnetization of each layer.

C. Arbitrary coupling strength

Now for any arbitrary interlayer magnetic coupling, one can still derive few useful analytical formulas allowing to infer, from an experimental torque curve, magnetic parameters characterizing such a magnetic tunnel junction-like system. This will be illustrated for the simple case where $\delta=0$ (no tilt angle) and $\beta=0$ (aligned case). In this situation, the torque curve will cross the α axis at $\alpha_0=0$ (no shift). Also, saturation is achieved, e.g., at $\alpha=0$, $\phi_{\mathrm{A}}=\phi_{\mathrm{B}}=0$, when the applied field is greater than the saturation field, this will be assumed in the subsequent analysis.

Let us first take the case of a coupled trilayer system (thus no exchange anisotropy H_E). An example of a torque curve for such a system is shown in Fig. 5 (dashed line). Let $s(J,0)$ be the torque curve slope at $\alpha=0$, for $H_E=0$ and arbitrary coupling. By measuring $s(J,0)$, it is found that the effective coupling parameter J_{eff} [equal to (J_1+2J_2) for this saturated system case] can be given by the following relation:

$$J_{\text{eff}} = ab\frac{H[PH+(a+b)H_{\text{A}}H_{\text{B}}]-s(J,0)(H+H_{\text{A}})(H+H_{\text{B}})}{s(J,0)[(a+b)H+P]-(a+b)PH}, \tag{28}$$

where $P=aH_{\text{A}}+bH_{\text{B}}$. Now, the spin valve system will be considered; the corresponding torque curve is shown in Fig. 5 (solid line). Assuming J_{eff} to be known from the trilayer system, Eq. (28) (experimentally this is done by growing at the same time the trilayer and the MTJ system, thus coupling can be assumed to be the same for both samples). The difference in the curve slopes (see for example at $\alpha=0$ in Fig. 5) is then due to the effect of H_E. Noting $s(J,H_E)$ and $s(J,0)$ the slopes at $\alpha=0$ for the spin valve and the coupled trilayer, respectively, then H_E can be derived by

$$H_E = \left(\frac{1}{a}\right)\frac{\{ab(H+H_{\text{A}})(H+H_{\text{B}})+J_{\text{eff}}[(a+b)H+P]\}[s(J,0)-s(J,H_E)]}{b[s(J,H_E)(H+H_{\text{B}})-H(aH+P)]+J_{\text{eff}}[s(J,H_E)-(a+b)H]}. \tag{29}$$

For uncoupled layers ($J_{\text{eff}}=0$), Eq. (29) becomes

$$H_E = \frac{(H+H_{\text{A}})(H+H_{\text{B}})[s(0,0)-s(0,H_E)]}{s(0,H_E)(H+H_{\text{B}})-aH(H+H_{\text{A}})-bHH_{\text{B}}}. \tag{30}$$

While for very strong coupling, H_E will be given by

$$H_E = \frac{[H(a+b)+aH_{\text{A}}+bH_{\text{B}}][s(\infty,0)-s(\infty,H_E)]}{a[s(\infty,H_E)-(a+b)H]}. \tag{31}$$

For strong applied field H and for any coupling strength, H_E will be simply given by

$$H_E = \frac{s(J,H_E)-s(J,0)}{a}. \tag{32}$$

Recall that $a=t_{\text{A}}M_{\text{A}}$ and the torque curve to be considered here is the torque per unit area.

Finally, when the coupling is antiferromagnetic, then one can derive analytical relations giving separate values of the bilinear J_1 and biquadratic J_2 coupling parameters from the measure of torque curves (note that generally a J_{eff} value is experimentally derived). If the applied field H_1 is greater than the saturation field, then at $\alpha=0$, $\phi_{\text{A}}=\phi_{\text{B}}=0$, let s_1 be the slope of the corresponding torque curve at $\alpha=0$. If the applied field H_2 is lower than the critical field (field below which the two magnetizations are antiparallel under the antiferromagnetic coupling) then at $\alpha=0$, $\phi_{\text{A}}=0$ and $\phi_{\text{B}}=\pi$ (assuming $a>b$ so that $\mathbf{M}_{\text{A}}$ will tend to the field direction while $\mathbf{M}_{\text{B}}$ will be opposite to it), let s_2 be the slope of the corresponding torque curve at $\alpha=0$. Then after some transformations J_1 and J_2 are given by

$$J_1 = \frac{(f_{1+})+(f_{2-})}{2} \tag{33}$$

and

$$J_2 = \frac{(f_{1+})-(f_{2-})}{4}, \tag{34}$$

where

$$f_{i\pm} = \frac{ab\{PH_i^2+(a\pm b)H_iH_{\text{A}}H_{\text{B}}-s_i(H_i+H_{\text{A}})(H_i\pm H_{\text{B}})\}}{H_i(a\pm b)(s_i-P)+s_iP}, \tag{35}$$

where $i=1,2$. Here for simplicity H_E is taken to be zero, generalizing to an arbitrary value of H_E is straightforward. Note that generally J_1 and J_2 cannot be separately measured, an effective J_{eff} (equal to $J_1 \pm 2J_2$) is instead derived. In some particular situations,[20,36,37] methods have been worked out to derive separate values of J_1 and J_2. The present method can be used to derive these parameters, in a straightforward and unique manner, by means of the analytical expressions and measurable quantities from torque curves.

V. CONCLUSION

Torque curves have been investigated for a series of multilayer systems, a single ferromagnetic layer with tilted axis anisotropy, an exchange coupled bilayer thin film, and a magnetic tunnel junction-like system (or spin valve). The analytical expressions pertaining to the torque curves allow to derive, in a straightforward manner, different magnetic parameters characterizing these systems. Some general relations involving features from torque curves have been obtained. It was found that for a strong coupling, the MTJ-like system behaves as an exchange coupled (F)/(AF) bilayer with effective exchange and anisotropy fields. In general, these characteristic fields have a nonlinear relationship as a function of the individual layer anisotropy fields. Moreover with the right choice of the magnetization and thickness of the ferromagnetic layers, one may tune the value of the exchange bias field of the whole system.

[1] S. S. P. Parkin, Advanced Research Workshop, Nanostructured Magnetic Materials and their Applications (NMMA), Istanbul, Turkey, July 1–4, 2003 (unpublished).
[2] S. S. P. Parkin, K. P. Roche, M. G. Samant, P. M. Riche, and R. B. Beyers, J. Appl. Phys. **85**, 5828 (1999).
[3] Y. Huai, J. Zhang, G. W. Anderson, P. Rana, and S. Funada, J. Appl. Phys. **85**, 5528 (1999).
[4] W. H. Meiklejohn and C. P. Bean, Phys. Rev. **102**, 1413 (1956); **105**, 904 (1957).
[5] B. Heinrich, Can. J. Phys. **78**, 161 (2000).
[6] T. J. Moran, J. Nogués, D. Lederman, and I. K. Schuller, Appl. Phys. Lett. **72**, 617 (1998).
[7] Y. Ijiri, J. A. Borchers, R. W. Erwin, S. H. Lee, P. J. van der Zaag, and R. M. Wolf, Phys. Rev. Lett. **80**, 608 (1998).
[8] R. D. McMichael, M. D. Stiles, P. J. Chen, and W. F. Egelhoff, Jr., Phys. Rev. B **58**, 8605 (1998).
[9] D. Spenato and S. P. Pogossian, J. Magn. Magn. Mater. **285**, 79 (2005).
[10] Kentaro Takano, R. H. Kodama, A. E. Berkowitz, W. Cao, and G. Thomas, Phys. Rev. Lett. **79**, 1130 (1997).
[11] H. Xi, K. R. Mountfield, and R. M. White, J. Appl. Phys. **87**, 4367 (2000).
[12] V. Ström, B. J. Jönsson, K. V. Rao, and D. Dahlberg, J. Appl. Phys. **81**, 5003 (1997).
[13] T. J. Moran, J. Nogués, D. Lederman, and I. K. Schuller, Appl. Phys. Lett. **72**, 617 (1998).
[14] H. Xi and R. M. White, J. Appl. Phys. **86**, 5169 (1999).
[15] A. Layadi, J. Appl. Phys. **90**, 4651 (2001).
[16] A. Layadi, Phys. Rev. B **66**, 184423 (2002).
[17] P. Grünberg, Acta Mater. **48**, 239 (2000).
[18] M. Rührig, R. Schäfer, A. Hubert, R. Mosler, J. A. Wolf, S. Demokritov, and P. Grünberg, Phys. Status Solidi A **125**, 635 (1991).
[19] J. C. Slonczewski, J. Magn. Magn. Mater. **150**, 13 (1995).
[20] C. Chesman, M. A. Lucena, M. C. de Moura, A. Azevedo, F. M. de Aguiar, S. M. Rezende, and S. S. P. Parkin, Phys. Rev. B **58**, 101 (1998).
[21] B. Heinrich, J. F. Cochran, T. Monchesky, and R. Urban, Phys. Rev. B **59**, 14520 (1999).
[22] B. Heinrich, M. Kowalewski, and J. F. Cochran, Can. J. Chem. **76**, 1595 (1998).
[23] S. M. Rezende, C. Chesman, M. A. Lucena, A. Azevedo, F. M. de Aguiar, and S. S. Parkin, J. Appl. Phys. **84**, 958 (1998).
[24] M. E. Filipkowski, C. J. Gutierrez, J. J. Krebs, and G. A. Prinz, J. Appl. Phys. **73**, 5963 (1993).
[25] Z. Zhang, L. Zhou, P. E. Wigen, and K. Ounadjela, Phys. Rev. B **50**, 6094 (1994).
[26] A. Layadi, J. Magn. Magn. Mater. **266**, 282 (2003).
[27] A. Layadi, Phys. Rev. B **72**, 024444 (2005).
[28] A. Layadi, J. Appl. Phys. **100**, 083904 (2006).
[29] J. Zabloudil, L. Szunyogh, U. Pustogowa, C. Uiberacker, and P. Weinberger, Phys. Rev. B **58**, 6316 (1998).
[30] C. Sommers, J. Zabloudil, C. Uiberacker, P. Weinberger, and L. Szunyogh, Phys. Rev. B **58**, 5539 (1998).
[31] R. Allenspach, M. Stampanoni, and A. Bischof, Phys. Rev. Lett. **65**, 3344 (1990).
[32] L. Udvardi, R. Kiraly, L. Szunyogh, F. Denat, M. B. Taylor, B. L. Györffy, B. Ujfalussy, and C. Uiberacker, J. Magn. Magn. Mater. **183**, 283 (1998).
[33] J. O. Artman, IEEE Trans. Magn. **21**, 1271 (1985).
[34] A. Layadi, J. Magn. Magn. Mater. **192**, 353 (1999).
[35] A. Layadi, J. Magn. Magn. Mater. **219**, 294 (2000).
[36] H. J. Elmers, G. Liu, H. Fritzsche, and U. Gradmann, Phys. Rev. B **52**, R696 (1995).
[37] A. Layadi, Phys. Rev. B **69**, 144431 (2004).